	ARTS IN MOTION CHARTER SCHOOL| 12th AP Calculus AB CURRICULUM MAP				
Projects	**Essential Questions**	**Enduring Understandings**	**Math Concepts**	**CCSS**	**Final Product**
Function Behavior & LImits	● How close can two points get together without touching?	● Limits allow us to find values of instantaneous rates of change.	● Continuity ● Limits	● NGSS.HS.A-APR.3 ● NGSS.HS.N-VM.6 ● NGSS.HS.N-VM.7 ● NGSS.HS.N-VM.8 ● NGSS.HS.N-VM.9 ● NGSS.HS.N-VM.10 ● NGSS.HS.N-VM.11 ● NGSS.HS.N-VM.12	● Books of Limit
Meaning & Computation of Derivatives	● How do we mathematically represent change?	● Derivatives are "slope functions" that allow us to describe how (most) functions are changing at every point	● Analyze Derivatives ● Continuity ● Derivative as Limit ● Derivatives of Functions	●	● Multiple Choice Assessment
Graphs and Applications of Derivatives	● What do the derivatives of functions tell us about the functions themselves? ● What types of problems do an understanding of derivatives and rates allow us to solve?	● The derivative represents the rate of change of a function and can be used to find zeroes and intervals of increase and decrease, a very helpful fact which can be applied to a variety of situations.	● Analyze Derivatives ● Interpret Derivatives	●	● Derivatives FRQ
Population Dynamics	● What is the most appropriate way to model data? ● What are the significant features of a graph?	● Setting the second derivative equal to zero provides information about the concavity of a graph. ● The first and second derivative of a function can be represented graphically and verbally in addition to the standard numerical method.	● Comparing/ Contrasting ● Hypothesizing ● Identifying Patterns and Relationships ● Introduction and Conclusion ● Making Connections & Inferences ● Modeling ● Organization (Transitions, Cohesion, Structure)		● Performance task

Meaning Computation of Antiderivatives	• What is the opposite of a derivative (i.e. what functions has ____ as its derivative)?	• Integrating a derivative of a function can tell you how the function itself is growing and changing.	• Area Under Curves • Fundamental Theorem of Calculus		• Riemann Sums: Performance Assessment • Meaning and Computation of Antiderivatives Performance Task
Techniques of Integration	• How do rates accumulate?	• Integrating a derivative of a function can tell you how the function itself is growing and changing.	• Accumulating Rates • Fundamental Theorem of Calculus		• Motion Modules Performance task • Techniques Integration Performance Task
Area and Volume	• How can integrals be used to find areas and volumes of more complicated figures?	• We can find the volume of almost shape we can imagine!	• Area & Volume		• Area and Volume Performance Task
Vivacious Volumes	• How can we apply the fundamental theorem of calculus to analyze objects around us?	• Calculus can be used to compute infinite sums, which allows us to determine exact measurements (such as volume) where basic analysis/manipulations would only lead to approximations.	• Identifying Patterns and Relationships • Justifying / Constructing an Explanation • Modeling • Multimedia in Written Production • Norms / Active Listening • Oral Presentation		• Performance Task
Differential Equations	• How can we solve equations defined by the derivative with 2 variables?	• Some equations that contain 2 variables that cannot be directly integrated can be manipulated so that they can be. • There is a difference between a 'general solution' which contains all possible solutions and a 'particular solution' found by establishing initial conditions.	• Differential Equations		• Traditional Performance Assessment

ARTS IN MOTION CHARTER SCHOOL|12th AP Calculus AB UNIT PLAN	
Project	• Function Behavior and Limits
Suggested Time	• 3 Weeks
Essential Questions	• How close can two points get together without touching?
Enduring Understandings	• Limits allow us to find values of instantaneous rates of change.
Math Concepts	• Continuity • Limits
Focus Areas	• 1) Graphs 1: Limits and Continuity
CCSS	•
Checkpoints	• Limits at Removable and Jump Discontinuities • One-Sided Limits and Other Limit Properties • Algebraic Limits • Limits at Infinity Asymptotic Behavior • Continuity
Final Product	• Books of Limit (See attached Sample)

ARTS IN MOTION CHARTER SCHOOL| 12th AP Calculus AB LESSON PLAN					
Project	• Function Behavior and Limits	**Essential Questions**	• How close can two points get together without touching?	**Final Product**	• Books of Limit

Checkpoint	• Limits at Removable and Jump Discontinuities (See attached Sample)
Math Concepts	• Continuity • Limits
Objective	• Identifying limits from graphs and tables. Identifying different points of discontinuity (and unbounded behavior) from tables and graphs. Estimating limits numerically by plugging points in closer and closer to the desired value to see how the values approach the limit.
Activities	• Warm Up and Notes: Limits at Removable and Jump Discontinuities (See attached Sample) • Task Card: Limits at Removable and Jump Discontinuities
Resources	• Limit Matching Lab (linked)
Assessment	• Performance task assessment using cognitive skills (See attached Sample)

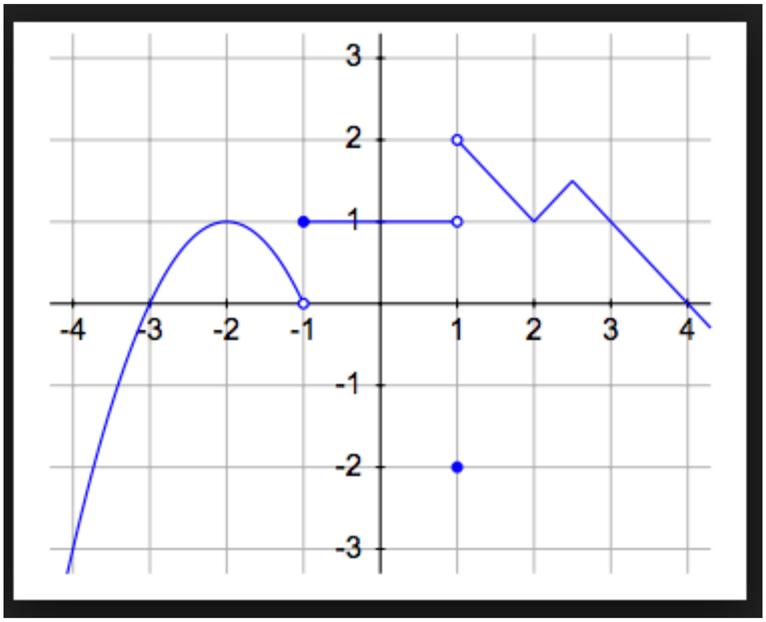
Function Behavior and Limits

CheckPoint: Limits at Removable and Jump Discontinuities

Concept(s):

- ❑ Limits

Refer to the piecewise function in the image below:



Part A:
Name 3 locations on the function above where the limit would be equivalent to 1. Explain all of your choices.

Your Answer:

Part B:
Explain why the limit does not exist at x = 1 even though the function is defined at x = 1.

Your Answer:

Part C:
Besides x = 1, at what other x-values on the piecewise function does the limit not exist? Explain your choice(s).

Your Answer:

Part D:
A student in your class argues that the following statement is correct with regard to the function above:

$$\lim_{x \to -1} f(x) = 1$$

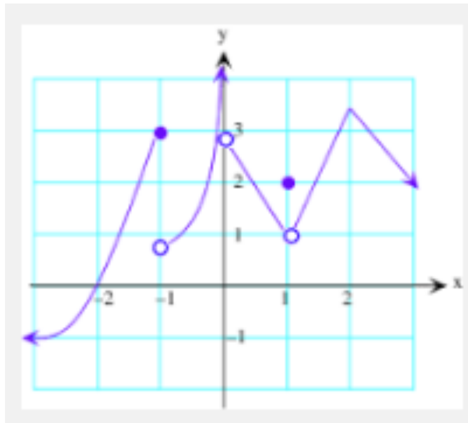
The student insists that this is the case because there is a closed point on the coordinate (-1, 1), so thus, the limit = 1. Explain where the student went wrong.

Your Answer:

Class 2 - Limits at Removable and Jump Discontinuities
WARM UP

Directions: use your notes from the pre-work to answer the questions below. Reminder: the pre-work from Class 1 was for you to watch and take notes on:
- "Introduction to Limits"
- "Limits from graphs: function undefined"

Look at the graph below:



(1) When x = -2, what is the y-value of the function?

(2) When x = -1, what is the y-value of the function?

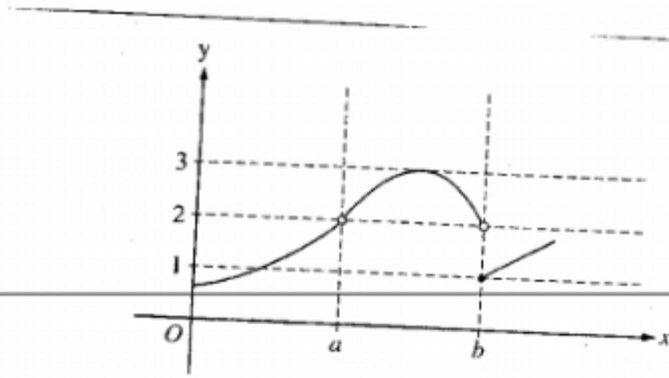
(3) How was determining the y-value of the function at x = -2 different than determining the y-value at x = -1? ***(Explain your answer)***

(4) Think back to the two videos that you watched for today's pre-work. How are "limits" of functions and "y-values" of functions different from each other?

Done Early?

Challenge Yourself: AP Calculus Limits Practice Problems (from the AP Exam)

Question 1



15. The graph of the function f is shown in the figure above. Which of the following statements about f is true?

(A) $\lim_{x \to a} f(x) = \lim_{x \to b} f(x)$

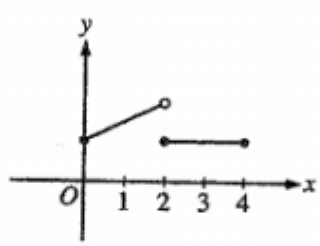
(B) $\lim_{x \to a} f(x) = 2$

(C) $\lim_{x \to b} f(x) = 2$

(D) $\lim_{x \to b} f(x) = 1$

(E) $\lim_{x \to a} f(x)$ does not exist.

Question 2



Graph of f

77. The figure above shows the graph of a function f with domain $0 \le x \le 4$. Which of the following statements are true?

I. $\lim_{x \to 2^-} f(x)$ exists.

II. $\lim_{x \to 2^+} f(x)$ exists.

III. $\lim_{x \to 2} f(x)$ exists.

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

Class 2 - Limits at Removable and Jump Discontinuities

NOTES

Pre-work:

- "Introduction to Limits"
- "Limits from graphs: function undefined"

Limits Visually - what do limits look like?:

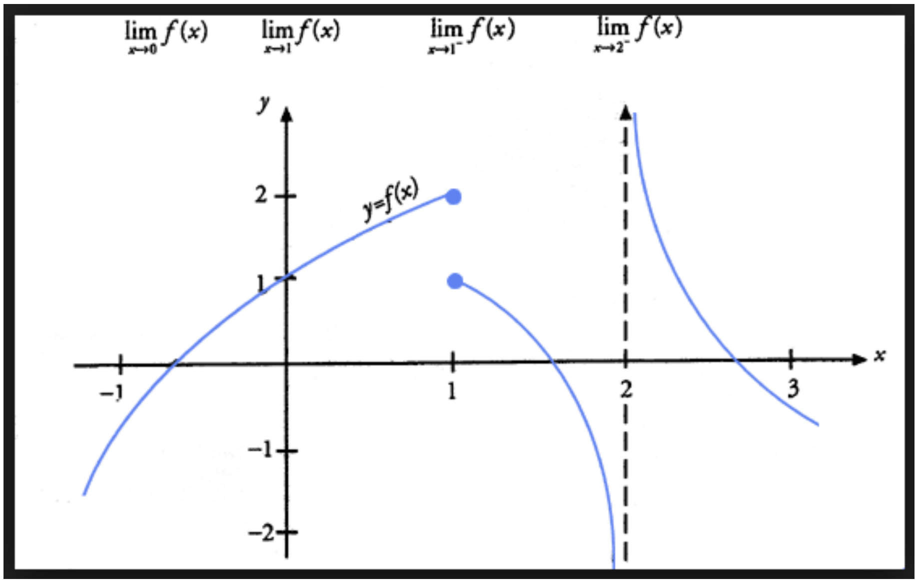
Example of a Point that does not have a limit:

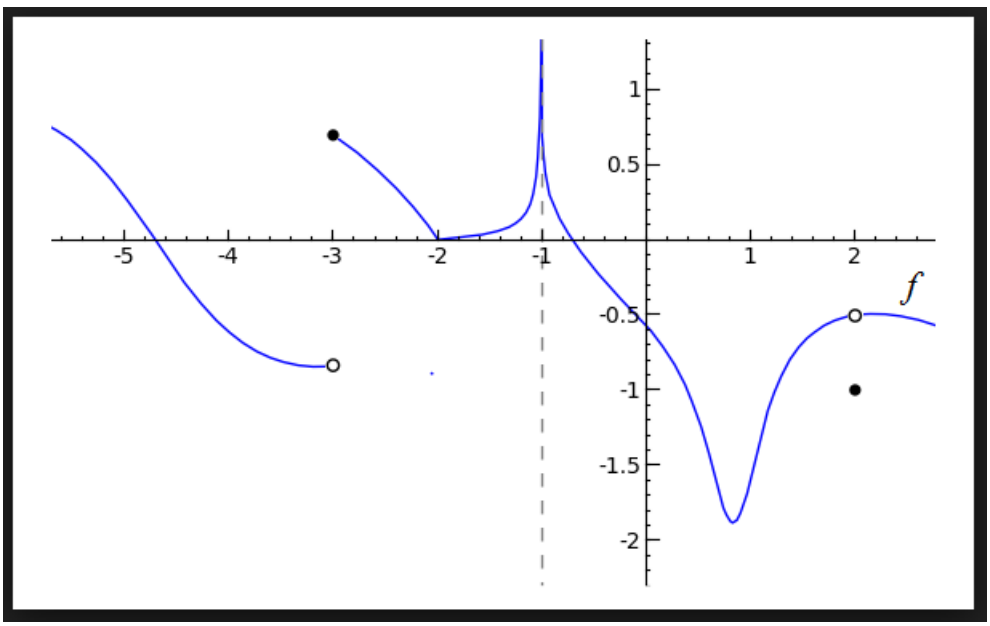
Important: We take limits at x-value points, not of entire functions. So, just because we see a discontinuity within a function, this does not imply that the entire function does not have a limit. It just means that the function doesn't have a limit at the x-value of the discontinuity.

Limit Notation:

Functions that have limits at all x-values	Functions that don't have limits of at all x-values.

A couple of Practice Problems for Playlist 1 [Optional]





What is the limit as x approaches -2?

What is the limit as x approaches 2?

What is the limit as x approaches -3?

Book of Limits

Description of Performance Task

You are to ***construct a piece-wise defined function***. Your piece-wise defined function should:

1) be defined by at least 12 different pieces, including at least one of each of the following functions:
    a. Constant
    b. Linear
    c. Absolute Value
    d. Quadratic
    e. Polynomial of degree 3 or higher
    f. Exponential
    g. Rational
    h. Irrational (Square Root or Cube Root)
    i. Sine/Cosine

2) be discontinuous in at least 8 places, including at least two instances of each of the following types of discontinuity:
    a. removable
    b. jump/step
    c. infinite/asymptotic

3) Bonus: Be continuous in at least 2 places *where one piece ends and another piece begins*, including one instance of each of the following types of continuity:
    a. continuous but not differentiable
    b. continuous and differentiable

Your final product will include two graphs:

- a hand-drawn graph using a poster board or presentation paper
- a computer-drawn graph using either Winplot or Geogebra or Demos

Book of Limits

Your final product will include a completed version of the following graphic organizer:

Numbered function	Algebraic representation of function	Type of function	Domain of function	Range of function
1.				
2.				
3.				
4.				
5.				
6.				
7.				
8.				
9.				
10.				
11.				
12.				

(you can add more rows if you use more than 12 functions)

Your final product will include a completed version of the following graphic organizer:

Book of Limits

At each x-coordinate where one piece ends and another piece begins, complete the following:

Limit	Verbal explanation (using mathematical notation) of why the limit has this value (or why the limit doesn't exist)
$\lim_{x \to [\,]} f(x) =$	
$\lim_{x \to [\,]} f(x) =$	
$\lim_{x \to [\,]} f(x) =$	
$\lim_{x \to [\,]} f(x) =$	
$\lim_{x \to [\,]} f(x) =$	
$\lim_{x \to [\,]} f(x) =$	
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$\lim_{x \to [\,]} f(x) =$	
$\lim_{x \to [\,]} f(x) =$	

Your final product will include a completed version of the following graphic organizer:

Lettered discontinuit	Type of discontinuity: **R**=removable	x-coordinate of	Use limit notation (including one-sided limits) to mathematically explain why the

Book of Limits

y	**J/S**=jump/step **I/A**=infinite/asymptoti c	discontinuity	type of discontinuity occurs at the indicated -coordinate.
A.			
B.			
C.			
D.			
E.			
F.			
G.			
H.			

Your final product will include a completed version of the following graphic organizer:

Lettered continuity	Type of continuity: CD=continuous/ differentiable CND=continuous	-coordinate of continuity	Use precise mathematical language **or** limit notation (including one-sided limits) to mathematically explain why the type of continuity occurs at the indicated -coordinate.

Book of Limits

	/ not differentiable		
P.			
Q.			

Your final product will include a completed version of the following graphic organizer:

Provide a list of the domains on which your function is increasing	Provide a list of the domains on which your function is decreasing

To show a numeric understanding of asymptotic behavior, identify a vertical asymptote on your graph.

Equation of vertical asymptote: ____

Book of Limits

Construct a table of values of the function for values of x getting closer and closer to the x-value of the vertical asymptote:

x	$f(x)$